As promised, in this post, I will discuss the questions I gave you in the last post. Let's apply the rules of exponents that we have learned. I will recap all the rules first and then we will proceed to the questions.

Rule 1: $a^m \times a^n = a^{(m + n)}$

Rule 2: $a^m / a^n = a^{(m - n)}$

Rule 3: $(a^m)^n = a^{mn}$

Rule 4: For any number a, $a^0 = 1$

Rule 5: a^m × b^m = (a × b)^m

which also implies that (a × b)^m = a^m × b^m

Rule 6: a^m / b^m = (a / b)^m

which also implies that (a / b)^m = a^m / b^m

Question 1: Given (1/4)^18 × (1/5)^n = 1/(2 × 10^35), find the value of n.

In this question, we have 4s and 5s on the left hand side and 2s and 10s on the right hand side. How will we equate the exponents if the bases are different? We cannot equate the exponents in that case. So what we need to do here is make bases same. Let's bring down all bases to prime number form.

(1/4)^18 × (1/5)^n = 1/(2 × 10^35)

(1/2^2)^18 × (1/5)^n = 1/(2 × (2 × 5)^35)

(I suggest you to write down these steps in your notebook. Since I cannot format the numbers properly in the editor, the above looks confusing even though it is very straight forward.)

Using Rule 6: (a / b)^m = a^m / b^m on left hand side, we get

1^18/(2^2)^18 × 1^n/5^n = 1/(2 × (2 × 5)^35)

1 to any power is 1. Next we use Rule 5: (a × b)^m = a^m × b^m on right hand side to get

1/(2^2)^18 × 1/5^n = 1/(2 × 2^35 × 5^35)

When we write 2, it implies that the power here is 1 i.e. 2 = 2^1. We substitute this on right hand side and use Rule 3: $(a^m)^n = a^{mn}$ on left hand side to get

1/2^36 × 1/5^n = 1/(2^1 × 2^35 × 5^35)

Now we use Rule 1: $a^m \times a^n = a^{(m + n)}$ on right hand side

1/2^36 × 1/5^n = 1/(2^36 × 5^35)

Notice that the power of 2 is the same on left and right hand side (as expected). The power of 5 on the left hand side is n and on the right hand side it is 35. For the equation to hold, n must be 35.

(Or you could have noticed right in the beginning that the only 5 on the left hand side is the one which is raised to the power of n and on the right hand side, you will get 5^35 since you will get 5 only from 10^35.)

Question 2: Is 5^m < 1000?

Statement 1: 5^(m + 1) > 3000

Statement 2: 5^(m – 1) = 5^m – 500

Solution:

First note that 5^4 = 625 and 5^5 = 3125 (even if you do not know this, it is fine. You don't need to calculate. Just observe that 5^5 = 625 ×5 will be greater than 3000 since 600 × 5 = 3000)

Statement 1: 5^(m + 1) > 3000

This means m + 1 is greater than 4 which implies that m is greater than 3. It doesn't mean that m + 1 is at least 5 because the question doesn't say that m has to be an integer. m + 1 could be 4.999 making m = 3.999. Since m can take values less than 4 and more than 4, 5^m could be less than 1000 and more than 1000. Hence this statement is not sufficient to say whether 5^m is less that 1000.

Statement 2: 5^(m – 1) = 5^m – 500

We re-arrange the given equation to get: 500 = 5^m – 5^(m – 1). Notice that on the right hand side, the terms are subtracted. So you cannot do anything with the exponents except take something common. What can you take common from 5^m and 5^(m – 1)? Obviously, 5^(m -1). This is not intuitive to many people. Let me show you an example first.

Say, the given equation is 4 × 5^a = 5^7 – 5^6

What can you take common from the right hand side? I think most of you will agree it is 5^6 i.e. the term with the smaller exponent. The right hand side will become 5^6 (5 – 1) = 4 × 5^6 so that we will get a = 6. Similarly, when you have the equation 500 = 5^m – 5^(m – 1), what can you take common from the right hand side? You can take 5^(m – 1) i.e. the term with the smaller exponent. Also, 500 = 125 × 4 = 5^3 × 4. So the equation becomes:

5^3 × 4 = 5^(m – 1) × (5 – 1)

5^3 = 5^(m – 1) (Cancel 4 from both sides)

Therefore, m – 1 = 3 giving us m = 4

Hence, 5^m = 625 which is less than 1000. This statement is sufficient to tell us that 5^m is less than 1000.

Answer (B).

These are some applications of Exponents that you need to understand well in order to comfortably work with exponents on the GMAT. There are many more interesting concepts and questions related to exponents so start practicing!